Current to frequency conversion in a Josephson circuit

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The voltage oscillations which occur in an ideally current-biased Josephson junction, were proposed to make a current standard for metrology [1]. We demonstrate similar oscillations in a more complex Josephson circuit derived from the Cooper pair box: the quantronium. When a constant current I is injected in the gate capacitor of this device, oscillations develop at the frequency $f_B = I/2e$, with e the electron charge. We detect these oscillations through the sidebands induced at multiples of f_B in the spectrum of a microwave signal reflected on the circuit, up to currents I exceeding 100 pA. We discuss the potential interest of this current to frequency conversion experiment for metrology.

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Exploiting the quantum properties of a current-biased Josephson junction to make a current standard suitable for metrology was proposed by Averin, Zorin and Likharev [1]. This system has a simple mechanical analog: the phase difference φ across the junction is equivalent to the position of a particle moving in the Josephson potential $-E_J \cos\varphi$, the voltage across the junction to the particle velocity, and the bias current I to an applied force. The dynamics of such a particle is well explained within the framework of the Bloch energy bands $\epsilon_i(p)$ formed by the eigenstates of the particle, with p its quasi-momentum [2]. It was predicted in particular that the voltage across the junction (particle velocity) oscillates at the Bloch frequency $f_B = I/2e$ [1]. These Bloch oscillations, which provide a direct link between time and current units, would be of fundamental interest for electrical metrology. However, it is extremely difficult to current-bias a junction because it requires to embed it in a circuit with a high impedance over a wide frequency range [3, 4]. On the other hand, it is easy to force Bloch oscillations [5] by imposing the quasi-momentum, that is the total bias charge Q delivered to the junction [1], by connecting it to a small gate capacitor C_g in series with a voltage source V_g, so that $Q = C_g V_g$. This scheme cannot impose a constant current, but can deliver alternatively two opposite values of the current $\mathrm{d}Q/\mathrm{d}t = \pm I$. In this Letter, we report experiments using this procedure and demonstrating oscillations at the Bloch frequency $f_B = I/2e$ in a Josephson circuit that allows their detection. Our setup, shown in Fig. 1a, is based on a modified Cooper pair box [6], the quantronium [7]. We show how this new current-to-frequency conversion method exploits the quantum properties of the circuit. We also discuss its interest in metrology of electrical currents, for which electron pumping [8, 9] and electron counting [10] have also been proposed.

The quantronium device [7, 11] is a split Cooper pair box that forms a loop including also a probe junction. The box island with total capacitance C is defined by two small junctions having Josephson energies $E_J(1+d)/2$ and $E_J(1-d)/2$, d being an asymmetry coefficient. The superconducting phase $\widehat{\theta}$ of this island, conjugated to the number $\widehat{N}$ of extra Cooper pairs inside, forms the single degree of freedom of the box [12]. The third larger junction with critical current I_0, in parallel with an added on-chip capacitor C_a, forms a resonator with plasma fre-

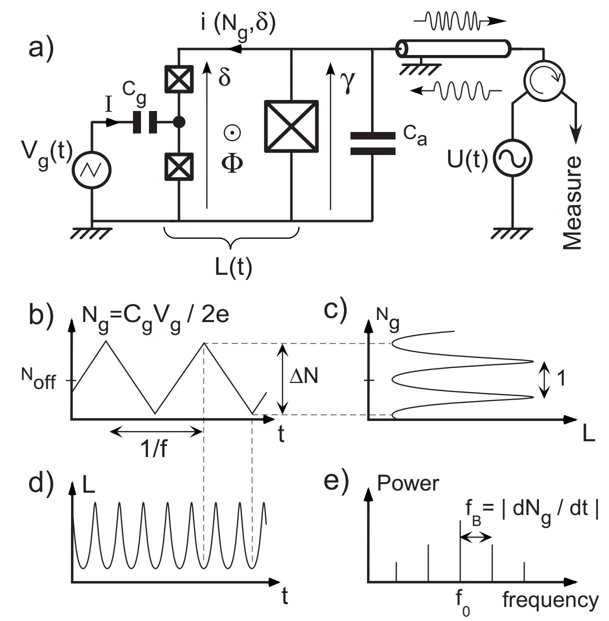

FIG. 1: Operating principle of the quantronium circuit for the production and detection of Bloch-like oscillations. The circuit(a) is a split Cooper pair box with a probe junction for the detection of the oscillations that develop when the gate charge is swept linearly. When the linear sweep is replaced by a triangular sweep (b) with extrema corresponding to symmetry points of the inductance modulation pattern (c), the time variations of the inductance (d) are the same as for a continuously increasing linear sweep. This modulation manifests itself as sidebands in the spectrum (e) of a microwave signal reflected onto the circuit.

quency f_p in the $1-2$ GHz range [13]. Since this frequency is always smaller than the box transition frequency, we treat the phase difference γ across the probe junction as a classical variable. The same thus holds for the phase difference $\delta = \gamma + \phi/\varphi_0$ across the two box junctions in series, with ϕ the magnetic flux applied through the loop, and $\varphi_0 = \hbar/2e$. The control parameters of the split-box are δ and the reduced gate charge $N_g = C_g V_g/2e$, with C_g the island gate capacitance, and V_g the gate voltage. The Hamiltonian of the box writes

$$\widehat{H} = E_C(\widehat{\mathbf{N}}-N_g)^2 - E_J \cos\frac{\delta}{2}\cos\widehat{\theta} + dE_J \sin\frac{\delta}{2}\sin\widehat{\theta}\ , \quad (1)$$

with $E_C = (2e)^2/2C$.

The eigenenergies $\epsilon_i(N_g,\delta)$ vary periodically with N_g (period 1) and δ (period 2π) [11]. The experiment consists in imposing a linear variation of the reduced quasi-momentum N_g that induces a periodic evolution of the quantum state along the first Bloch band $\epsilon_0(N_g,\delta)$, at the Bloch frequency $f_B = \mathrm{d}N_g/\mathrm{d}t = I/2e$. Therefore, the current $i(N_g,\delta) = \varphi_0^{-1}\partial\epsilon_0(N_g,\delta)/\partial\delta$ through the two small junctions, the associated effective inductance for small phase excursions

$$L(N_g,\delta) = \varphi_0^2\left(\frac{\partial^2\epsilon_0(N_g,\delta)}{\partial\delta^2}\right)^{-1}, \quad (2)$$

and hence the admittance $Y(\omega) = j\left[C\omega - I_0/\varphi_0\omega - 1/L(N_g,\delta)\omega\right]$ as seen from the measuring line, vary periodically. We measure this admittance $Y(\omega)$ by microwave reflectometry, as for the RF-SET [14]. In our experiment, we apply a triangular modulation of the gate signal centered on $N_g = N_{\mathrm{off}}$, with peak to peak amplitude ΔN, and with frequency f_g corresponding to $\mathrm{d}N_g/\mathrm{d}t = \pm I/2e$. Due to the symmetry properties of the quantum states with respect to N_g, the inductance varies as for a linear sweep, as shown in Fig. 1, provided that the extremal values of N_g are integer or half-integer, with a Bloch frequency $f_B = 2\Delta N f_g$. In order to obtain the largest gate charge modulation of the inductance, the phase is adjusted at $\delta \approx \pi$ with the flux ϕ. When a small microwave signal at frequency f_0 is sent on the measuring line, the periodic modulation of the reflection factor yields sidebands in the spectrum of the reflected signal [14], shifted from the carrier by multiples of f_B, and called Bloch lines [15]. Due to the periodic excitation, the stationary outgoing amplitude can be written as a series:

$$v_{out}(t) = \sum_k v_k \exp[2i\pi(f_0 + kf_g)t]. \quad (3)$$

The circuit equations and the loop-current expression [7, 11] allow to calculate all sideband amplitudes v_k, which get smaller and become asymmetric ($v_{-k} \neq v_k$) when the sideband frequencies depart from the resonance.

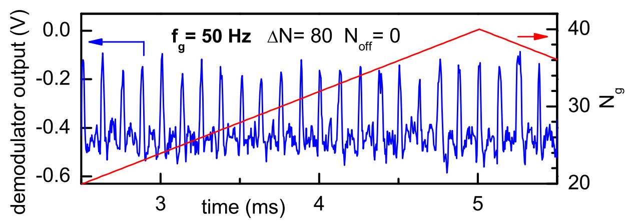

FIG. 2: (Color online) Demodulated output signal (left scale) recorded with a 300 Hz - 30 kHz bandwidth during a 3 ms time window, when a triangular wave voltage corresponding to a Bloch frequency $f_B = 8$ kHz is applied to the gate (right scale). Each period corresponds to the injection of one extra Cooper pair.

The sample was fabricated using electron-beam lithography [7, 11] and aluminum deposition and oxidation. In order to avoid quasiparticle poisoning, the island was made thinner than the leads (13 nm and 42 nm respectively), and gold quasiparticle traps were used [16]. In the present experiment, a sizeable asymmetry d was introduced on purpose in order to maintain a large gap $G_0 = \epsilon_1 - \epsilon_0$ at $(N_g = 1/2, \delta = \pi)$, which avoids microwave driven transitions towards excited bands. The sample was placed in a sample-holder fitted with microwave transmission lines, at a temperature $T \simeq 30$ mK. The gate was connected to a 250 MHz-bandwidth RF line. The microwave signal, after reflecting on the probe junction, went through 3 circulators before being amplified by a cryogenic amplifier with noise temperature $T_N = 2.2$ K, and a room temperature amplifier. The signal was then either demodulated with the CW input signal, or sent to a spectrum analyzer. In the latter case, the applied power was ≈ -132 dBm, corresponding to phase excursions smaller than ± 0.1 rad, and the total measurement gain was ≈ 88 dB. As a function of N_g and δ, the plasma resonance varied in the range $1.11-1.21$ GHz, slightly below the circulator bandwidth, which yielded an extra attenuation of the signal due to a spurious interference with the leakage signal through circulator [17]. Fitting the variations of the resonance yielded $f_p = 1.19$ GHz, $Q \approx 17$, $E_C = 1.42 \pm 0.2$ $\mathrm{k_B}$K, $E_J = 2.88 \pm 0.2$ $\mathrm{k_B}$K, and $d = 0.15 \pm 0.03$ leading to $G_0 \approx 7hf_0$.

The reflected signal demodulated with the carrier is shown in Fig. 2 for a triangular gate voltage corresponding to $f_B = 8$ kHz. Due to noise, such time-domain measurements could only be performed within a 100 kHz bandwidth [18]. In the following, the reflectometry spectra are taken with a 1 Hz bandwidth resolution.

A series of spectra recorded at $f_0 = 1.14$ GHz with $f_g = 200$ Hz, and taken with progressively tuned gate sweep signal amplitude and offset is shown in Fig. 3: when ΔN and N_{off} are tuned as sketched in Fig. 1, the spectrum consists only of Bloch lines, as predicted, with

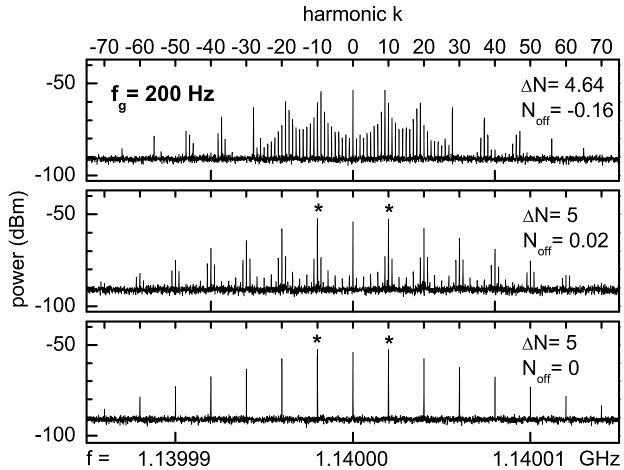

FIG. 3: Spectrum of a reflected CW signal at 1.14 GHz for different triangular wave gate modulation patterns, at frequency $f_g = 200$ Hz. The spectrum consists of sidebands shifted by kf_g from the carrier. Progressive tuning of the amplitude ΔN and of the offset $N_{\rm off}$ yields a spectrum consisting only of Bloch lines shifted from the carrier by a multiple of $f_B = 2\Delta N f_g$. The Bloch lines of order 1 ($k = \pm 2\Delta N$) are marked by an asterisk.

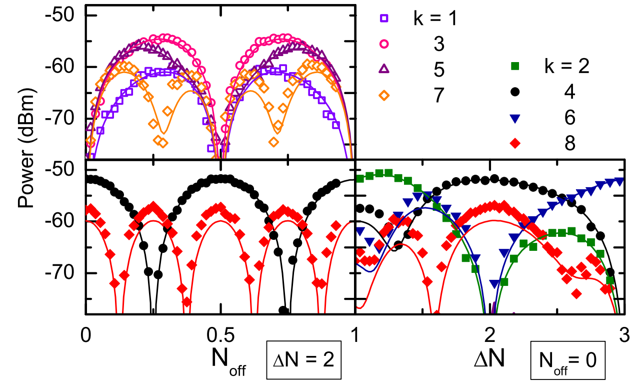

FIG. 4: (Color online) Comparison of the measured (symbols) and calculated (lines) sideband amplitudes as a function of the sweep offset $N_{\rm off}$ and amplitude ΔN, for $f_g = 1$ kHz. Left panels: Offset dependence for $\Delta N = 2$; the observed sidebands correspond as predicted to odd multiples of f_g (top) and to the Bloch line $k = 4$ and its harmonics $k = 4n$, with period $1/(2n)$ in $N_{\rm off}$ (bottom). Right panel: amplitude dependence for $N_{\rm off} = 0$. The Bloch line corresponds to harmonic 4 at $\Delta N = 2$, and to harmonic 6 at $\Delta N = 3$. Calculated curves were shifted by 45 dB (estimated value was 44 dB) to best match the experimental Bloch line of order 1.

linewidth limited by the spectrum analyzer.

An example of comparison between the measured and predicted sideband amplitudes when $N_{\rm off}$ or ΔN is varied is shown in Fig. 4 for $f_g = 1$ kHz. The measured amplitudes are well accounted for by the solution of Eq. 2, but for the carrier, which suffers from the spurious interference effect already mentioned [17]. Although the overall agreement for the sidebands, and in particular the cancelation of some of them at particular offsets and amplitudes, demonstrate the phase-coherence of the measured signal, it does not prove that the quantronium undergoes a perfect coherent adiabatic evolution of its ground-state while N_g and δ are varied: incoherent excitation/de-excitation processes are for instance not excluded. Because of the opposed inductance modulation in the excited state, they would only reduce the amplitude of the Bloch lines in proportion of the time spent in this state.

Spectra obtained at larger frequencies f_g and larger amplitudes ΔN, corresponding respectively to currents and Bloch frequencies $I = 32$ pA, $f_B = 100$ MHz, and $I = 130$ pA, $f_B = 408$ MHz, are shown in Fig. 5. These results demonstrate that Bloch oscillations persist at Bloch frequencies larger than the resonator bandwidth $f_p/Q \approx 70$ MHz, even though Bloch lines become weaker. The successful current to frequency conversion performed up to currents $I > 100$ pA using Bloch oscillations is the main result of this work. However, the amplitudes of the Bloch lines at these high frequencies are smaller than predicted by the model, and additional sidebands are present, which we attribute to the rounding of the gate triangular wave signal at its turning points, and to drifts of the gate-charge due to background charge noise. In principle, the theoretical maximum current is limited by two fundamental phenomena. First, the conversion mechanism requires $f_B \ll f_0$, with $f_0 \ll G_0/h$ to avoid multi-photon excitations. Second, the current must be low enough to avoid Zener transitions at $N_g = 1/2$ where the gap is minimum. For the present experiment, the Zener probability $p_Z = \exp(-\pi^2 {G_0}^2/hsf_B) \approx \exp(-13 \text{ GHz}/f_B)$, with s the slope of $\epsilon_1(N_g) - \epsilon_0(N_g)$ away from $N_g = 1/2$, is negligible.

An important application of these experimental results could be to establish a direct link between a DC current and a frequency through the Bloch frequency $f_B = I/2e$, in order to close the triangle of quantum metrology [19]. Such an experiment would aim at measuring the current I_H passing through a Quantum Hall bar device in terms of a rate $\dot{N}_H$ of transferred Cooper pairs, in order to check the consistency of the Quantum Hall effect (QHE) with the AC Josephson effect. Using this latter effect, the Hall voltage $V_H = (h/e^2)I_H$ across a QHE bar can indeed be related to a frequency f_H through the relation $(h/e^2)I_H = (h/2e)f_H$. If the description of both QHE and Josephson experiments is exact, one predicts $\dot{N}_H = f_H/4$. A consistency check of this relation at the 10^{-8} level is presently a major goal in metrology because, in conjunction with a metrological realization of the mass unit by a Watt-balance experiment [20], it would provide a serious basis for a redetermination of the SI unit system in terms of electrical experiments involving only funda-

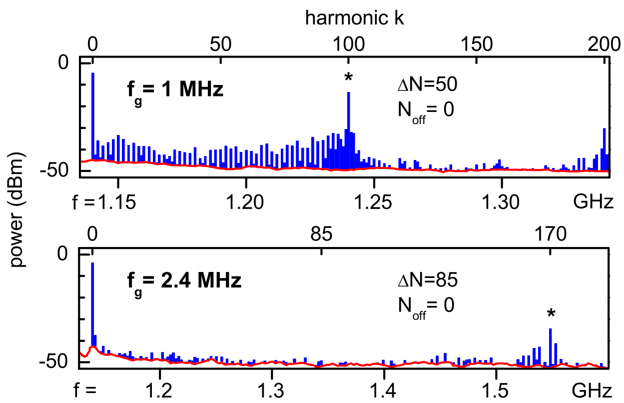

FIG. 5: (Color online) Amplitude of the sidebands at positive harmonics (top scale) of the gate frequency f_g. Top panel: $\Delta N = 50$, $f_g = 1$ MHz ($I = 32$ pA,$f_B = 100$ MHz) ; bottom panel: $\Delta N = 85$, $f_g = 2.4$ MHz ($I = 130$ pA, $f_B = 408$ MHz). The Bloch line of order 1 is marked by an asterisk. The continuous line is the noise level. We attribute the presence of non Bloch lines to the imperfections in the gate signal, and to charge noise.

mental constants. The large current $I_H \approx 1$ μA, needed for QHE experiments, can be transposed to a smaller range $0.1 - 1$ nA using topologically defined transformers [21]. This current range, which is still beyond reach of single electron pumps [8] or of direct electron counting experiments [10], can be accessed with the sluice Cooper pair pump [9], with Bloch oscillations in a single Josephson junction [3], or with the method demonstrated here provided it can be used with a true DC current. In the last two cases, the impedance of the current source as seen from the single junction or from the box needs to be larger than $R_Q = h/4e^2$ to preserve single Cooper pair effects, and temporal fluctuations have to be small enough to obtain narrow Bloch lines enabling an accurate measurement of their frequency. Using for instance a resistive bias yields a Bloch linewidth of the order of the Bloch frequency [3], due to thermal fluctuations of the self-heated bias resistor. Developing a suitable current source for charge injection is thus a challenging prerequisite to metrology experiments based on Bloch physics. High impedance dissipative linear Josephson arrays have already been used to demonstrate indirectly Bloch oscillations [4]. Combining ohmic, inductive, and Josephson elements, and possibly non-equilibrium cooling techniques [22], might provide an adequate low-noise high impedance.

In conclusion, we have demonstrated the conversion of a current $\pm I$ to a frequency $f_B = I/2e$ in a Josephson device biased through a small capacitor, through the production of ultra narrow sidebands in the spectrum of a reflected microwave signal. This new method, which reaches a current range $I > 0.1$nA, would be extremely appealing for metrology if operated with a DC current.

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